Geopotential and Geometric Altitude

The standard atmosphere is defined in terms of geopotential altitude. The idea behind this concept is that a small change in geometric altitude will make a change in gravitational potential energy per unit mass. To make this same change in potential energy at sea level requires a equivalent change in geopotential altitude. Mathematically, this is expressed as g dZ = G dH where H stands for geopotential altitude and Z stands for geometric altitude, g is the acceleration of gravity and G is the value of g at sea level. The value of g varies with altitude and is shown in elementary physics texts to vary as

$$\frac{g}{G} = \left(\frac{E}{Z+E}\right)^2$$

where E is the radius of the earth. So,

$$dH = \frac{g}{G}dZ = \left(\frac{E}{Z+E}\right)^2 dZ$$

and integrating yields

$$\int_0^H dH = \int_0^Z \left(\frac{E}{Z+E}\right)^2 dZ$$

$$H = \frac{EZ}{E+Z}$$

It follows that

$$Z = \frac{EH}{E-H}$$

While Z and H are virtually identical at low altitudes, you can calculate that Z = 86 km corresponds to H=84.852 km. (Use 6356 km for the radius of the earth). At this altitude, g is 0.9735 times the value at sea level.

If you don't like the definition of H as a differential, you can regard H=EZ / (E+Z) as the definition of H and then derive dH/dZ=g/G.

Last updated 21 January 2003 by Ralph Carmichael (ralph@pdas.com)
Public Domain Aeronautical Software
P.O. Box 1438
Santa Cruz CA 95061