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Practical transition function

In many cases a simple approach for probability calculations gives a range of a parameter – two critical values – outside which the probability is equal to zero and one. The simplest function connecting these values is a line. In some cases however it is desirable to smooth the first derivative at the junction points. In other cases it can even be desirable to smooth the second and so forth derivatives. In such case the simplest general family of function can be defined as:

$$s_{n+1}^{*}(x) = \frac{(2n+1)!}{n!^2} \int \left(x - x^2\right)^n dx$$

$$s_n^{*}(0) = s_n^{*}(1) - 1 = 0$$

because the simplest polynomial form giving zeros in 0 and 1 is $x(1-x)$, and the coefficient in front of the integral is set to match the condition. Below are the calculated forms for the first 4 orders which are shown in Figure 1.

$$s_1^{*}(x) = x$$

$$s_2^{*}(x) = 3x^2 - 2x^3$$

$$s_3^{*}(x) = 6x^5 - 15x^4 + 10x^3$$

$$s_4^{*}(x) = -20x^7 + 70x^6 - 84x^5 + 35x^4$$

$$\ldots$$

Then an arbitrary function F with critical values $F_1 = F(x_1)$ and $F_2 = F(x_2)$ is expressed in $[x_1, x_2]$ as:

$$F(x) = F_1 + (F_2 - F_1)\, s_n^{*}\left(\frac{x - x_1}{x_2 - x_1}\right)$$

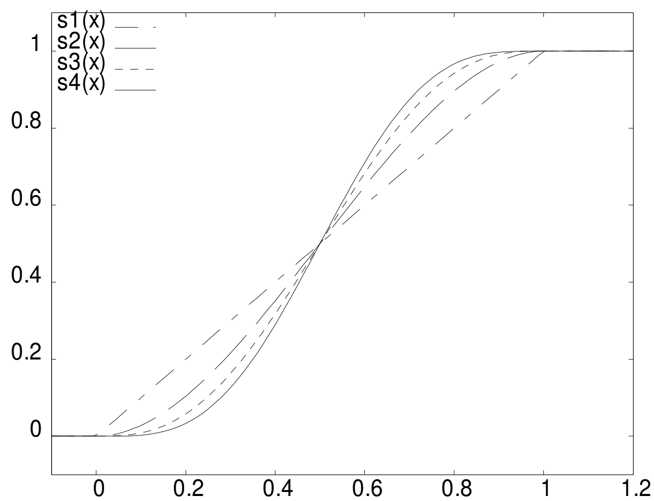

Figure 1 Practical transition function for 1, 2, 3, and 4th order.